Sirindhorn International Institute of Technology
Thammasat University
School of Information, Computer and Communication Technology

ECS315 2019/1 Part VII Dr.Prapun

13 Three Types of Random Variables

13.1. Review: You may recall[56] the following properties for cdf of discrete random variables. These properties hold for any kind of random variables.

(a) The cdf is defined as $F_X(x) = P[X \le x]$. This is valid for any type of random variables.

(b) Moreover, the cdf for any kind of random variable must satisfies three properties which we have discussed earlier:

CDF1 F_X is non-decreasing

CDF2 F_X is right-continuous

CDF3 $\lim_{x\to-\infty} F_X(x) = 0$ and $\lim_{x\to\infty} F_X(x) = 1$.

(c) $P[X = x] = F_X(x) - F_X(x^-) =$ the jump or saltus in F at x.

Theorem 13.2. If you find a function F that satisfies CDF1, CDF2, and CDF3 above, then F is a cdf of some random variable.

[56] If you don't know these properties by now, you should review them as soon as possible.

Example 13.3. Consider an input X to a device whose output Y will be the same as the input if the input level does not exceed 5. For input level that exceeds 5, the output will be saturated at 5. Suppose $X \sim \mathcal{U}(0, 6)$. Find $F_Y(y)$.

13.4. We can categorize random variables into three types according to its cdf:

(a) If $F_X(x)$ is piecewise flat with discontinuous jumps, then X is **discrete**.

(b) If $F_X(x)$ is a continuous function, then X is **continuous**.

(c) If $F_X(x)$ is a piecewise continuous function with discontinuities, then X is **mixed**.

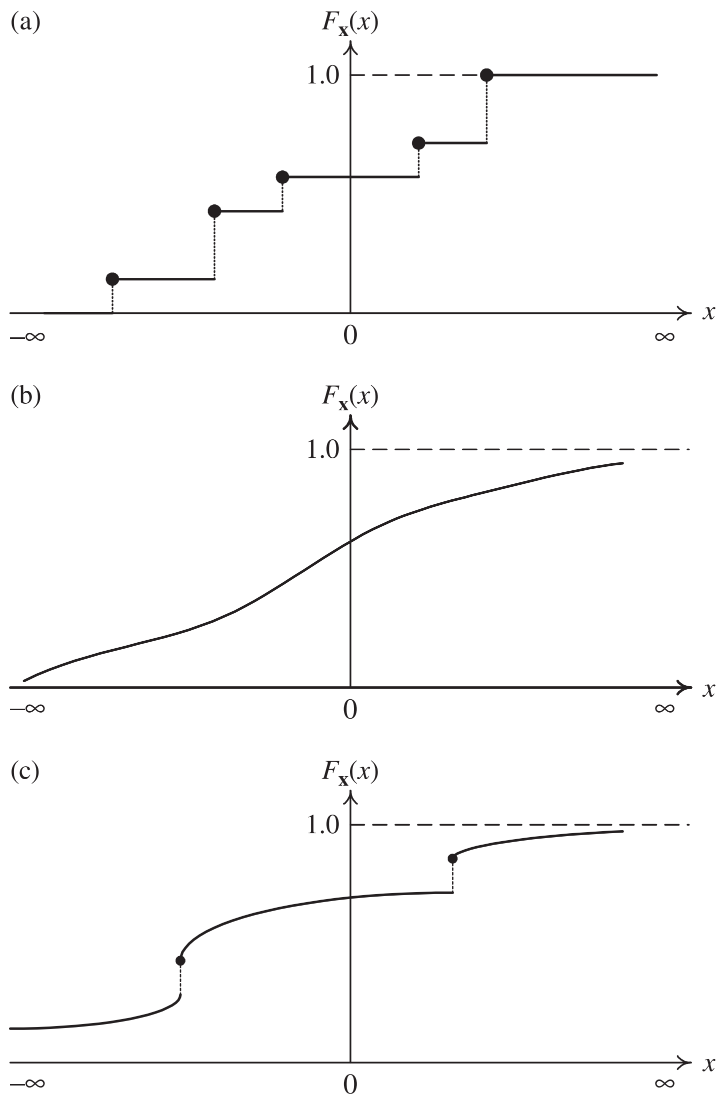

Figure 45: Typical cdfs: (a) a discrete random variable, (b) a continuous random variable, and (c) a mixed random variable [16, Fig. 3.2].

We have seen in Example 13.3 that some function can turn a continuous random variable into a mixed random variable. Next, we will work on an example where a continuous random variable is turned into a discrete random variable.

Example 13.5. Let $X \sim \mathcal{U}(0,1)$ and $Y = g(X)$ where

$$g(x) = \begin{cases} 1, & x < 0.6 \\ 0, & x \geq 0.6. \end{cases}$$

Before going deeply into the math, it is helpful to think about the nature of the derived random variable Y. The definition of $g(x)$ tells us that Y has only two possible values, $Y = 0$ and $Y = 1$. Thus, Y is a discrete random variable.

Example 13.6. In `MATLAB`, we have the `rand` command to generate $\mathcal{U}(0,1)$. If we want to generate a Bernoulli random variable with success probability p, what can we do?

Exercise 13.7. In `MATLAB`, how can we generate $X \sim \text{binomial}(2, 1/4)$ from the `rand` command?